Poster: Topology Properties of Ad Hoc Networks with Random Waypoint Mobility

Christian Bettstetter
Technische Universität München (TUM), Institute of Communication Networks, Munich, Germany
Christian.Bettstetter@ei.tum.de

ABSTRACT

Given is a wireless multihop network with n nodes moving according to the random waypoint model on a system area of size A. Each node has a fixed transmission range r_0. We derive a closed–form approximation for the critical (r_0, n)–pairs that are needed to keep the mobile network connected during at least $P(\text{con}) = 99\,\%$ of the time. In addition, we calculate the average degree μ experienced by a node during its movement and the node–to–node distance distribution $f_S(s)$.

Category and Subject Descriptors: C.2 [Computer-communication networks]: Network architecture and design — *wireless communication, network communication, network topology*

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1. INTRODUCTION

Publications dealing with the analytical investigation of connectivity in wireless multihop networks usually assume that nodes are distributed according to a (static) uniform spatial density. This paper studies the connectivity of *mobile* nodes with *non–uniform* spatial density. We regard a network with n nodes moving according to the random waypoint (RWP) model [1] on a system area $\mathcal{A}$. Each node has the same transmission range r_0, and two nodes establish a link if they are located within distance r_0 of each other. We are interested in the analytical solution to the following problem: What is the minimum r_0 for given n, such that the mobile network is connected with high probability, say $P(\text{con}) = 99\,\%$? In other words: which (r_0, n)–pairs result in a connected network during at least $99\,\%$ of the time? On our way to the solution, we also study uniformly distributed nodes with consideration of border effects; a previous paper of the author [2] regarded only scenarios in which these effects can be ignored. Related work can be found in [3, 4] and references therein. In addition, we derive equations for the expected degree μ of RWP nodes and the probability density function (pdf) of the distance S between them. Both measures are important topology properties. For instance, the distance between two nodes influences the number of hops between them, which in turn has impact on the end–to–end delay of packet delivery.

Our study focuses on a disk with radius a and size $A = \|\mathcal{A}\| = a^2\pi$. When appropriate, we use polar coordinates $(r = \sqrt{x^2 + y^2}, \phi)$ and normalized variables $\hat{r}_0 = r_0/a$, $\hat{r} = r/a$, and $\hat{S} = S/a$.

2. RANDOM WAYPOINT MODEL

The RWP model is a frequently used mobility model in ad hoc networking research. According to this model, a node randomly chooses a destination point ('waypoint') in $\mathcal{A}$ and moves at constant speed in a straight line to this point. The node then rests for a certain time period (pause time T_p), chooses a new destination and speed, moves with constant speed to this destination, and so on. The destination points are chosen from a uniform distribution.

It has been observed in previous papers that the spatial node density resulting from a long–run RWP movement is non–uniform. The steady–state pdf of a node's location $\mathbf{X} = (X, Y)$ is given by $f_{\mathbf{X}} = q f_{\mathbf{X},p} + (1-q) f_{\mathbf{X},m}$, where $f_{\mathbf{X},p}$ is the density of all pausing nodes, given by a uniform pdf over $\mathcal{A}$, and $f_{\mathbf{X},m}$ is the pdf of all mobile nodes. In a circular area, the latter can be approximated by $f_{\mathbf{X},m} = f_{XY}(x,y) \approx \frac{1}{a^2\pi}(-\frac{2}{a^2}\, r^2 + 2)$ for $0 \le r \le a$ [5]. The pause probability [6] is $q = \frac{E\{T_p\}}{E\{T_p\}+E\{T\}}$, where $E\{T\}$ denotes the expected movement time between two waypoints. It is given by $E\{T\} = 0.905\, a/v$ for constant speed v on a disk.

3. NODE DEGREE AND DISTANCES

Let $\mathcal{C}_0(\mathbf{x})$ denote the area that a node at location $\mathbf{x}$ covers with its transmission power. The probability that a randomly chosen node is located within this coverage area is $P_0(\mathbf{x}) = \iint_{\mathcal{C}_0(\mathbf{x})} f_{\mathbf{X}} dA = \int_{y-r_0}^{y+r_0} \int_{x-\sqrt{r_0^2-(y'-y)^2}}^{x+\sqrt{r_0^2-(y'-y)^2}} f_{XY}(x', y')\; dx' dy'$. The expected degree of a node at $\mathbf{x}$ is then $\mu(\mathbf{x}) = n P_0(\mathbf{x})$.

For uniformly distributed nodes on a disk, we obtain $\mu_p(\mathbf{x}) = n\,\hat{r}_0^2$ for $0 \le \hat{r} \le 1 - \hat{r}_0$, and $\mu_p(\mathbf{x}) = \frac{n}{\pi}\; (\hat{r}_0^2 \, \arccos \frac{\hat{r}^2+\hat{r}_0^2-1}{2\,\hat{r}\,\hat{r}_0} + \arccos \frac{\hat{r}^2-\hat{r}_0^2+1}{2\,\hat{r}} - \frac{1}{2}\,\mathcal{S})$ for $1-\hat{r}_0 < \hat{r} \le 1$ with $\mathcal{S} = \sqrt{\hat{r}+\hat{r}_0+1}\,\sqrt{-\hat{r}+\hat{r}_0+1}\;\sqrt{\hat{r}-\hat{r}_0+1}\;\sqrt{\hat{r}+\hat{r}_0-1}$. Using the above $f_{\mathbf{X},m}$ yields $\mu_m(\mathbf{x}) = n(-2\hat{r}_0^2\,\hat{r}^2 + 2\hat{r}_0^2 - \hat{r}_0^4)$ for $0 \le \hat{r} \le 1 - \hat{r}_0$ and $\frac{n}{4\pi}\big[4\hat{r}_0^2\,\big(2\hat{r}^2 + \hat{r}_0^2 - 2\big) \arcsin \frac{\hat{r}^2+\hat{r}_0^2-1}{2\hat{r}\hat{r}_0} - 4 \arcsin \frac{\hat{r}^2-\hat{r}_0^2+1}{2\hat{r}} + \mathcal{S}\,\big(\hat{r}^2 + 5\hat{r}_0^2 - 3\big) + 2\pi(-2\hat{r}^2\hat{r}_0^2 + 2\hat{r}_0^2 - \hat{r}_0^4 + 1)\big]$ for $1-\hat{r}_0 < \hat{r} \le 1$. Combining the pause and mobility components gives $\mu(\mathbf{x}) = q\,\mu_p(\mathbf{x}) + (1-q)\,\mu_m(\mathbf{x})$.

From this, we can compute the average degree $\mu = \iint_{\mathcal{A}} \mu(\mathbf{x}) f_{\mathbf{X}} dA$ experienced by an RWP node during its entire movement. Proper integration and expansion into a Taylor series with respect to $\hat{r}_0$ yields $\mu \approx \frac{n\hat{r}_0^2}{3}\big((4-2q+q^2) - \frac{4}{\pi} q^2\,\hat{r}_0 - 3(1-q)\,\hat{r}_0^2\big)$ for $\hat{r}_0 \le 0.3$.

Let us now regard the distance S between two nodes. Its cumulative distribution is defined by $F_S(s) = P(S \le s) = \frac{\mu}{n}\,|_{r_0=s}$. Taking the derivate of $F_S(s)$ and performing normalization yields the pdf $f_{\hat{S}}(\hat{s}) = \frac{\hat{s}}{9\pi}\big[\,\big(6q^2 + (36\hat{s}^2 - 12)q - 36\hat{s}^2 + 24\big)\pi + \big(-12q^2 + (-72\hat{s}^2 + 24)q + 72\hat{s}^2 - 48\big) \arcsin \frac{\hat{s}}{2} + \big((-\hat{s}^5 + 7\hat{s}^3 - 15\hat{s})q^2 + (2\hat{s}^5 - 23\hat{s}^3 - 6\hat{s})q - \hat{s}^5 + 16\hat{s}^3 + 12\hat{s}\big)\sqrt{4 - \hat{s}^2}\,\big]$ with $\hat{s} = s/a$. Note that $f_S(s) = \frac{1}{a}\, f_{\hat{S}}(s/a)$. The expected distance is $E\{\hat{S}\} = (-5q^2 + 64q + 256)\,\frac{128}{14175\pi}$.

Using $q = 1$ or 0 gives us the expressions for uniform nodes or RWP nodes with no pause time, respectively. We observe that RWP mobility increases μ and decreases $E\{\hat{S}\}$.

4. CONNECTIVITY

A network is connected if and only if there is a path between each pair of nodes. The minimum range that is needed to obtain, with a certain probability p, a network with no isolated node is therefore a lower bound for the range that is required to achieve, with the same probability p, a

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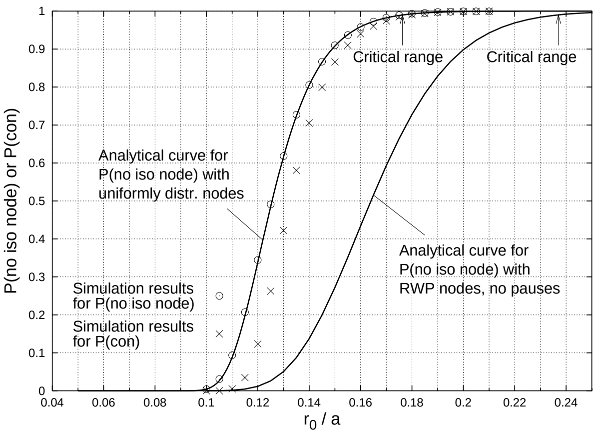

Figure 1: Probability that none of $n = 500$ nodes is isolated

connected network: $r_0\,(P(\text{con})\!=\!p, n) \geq r_0\,(P(\text{no iso node}) = p, n)$. We are especially interested in high connection probabilities $p\!=\!0.99$.

In the following, we assume that $\hat{r}_0 \leq 0.3$ and $n > 100$, such that binomial distributions can be well approximated by Poisson distributions. The probability that a node at $\mathbf{x}$ is isolated is $P(\text{iso} \mid \mathbf{x}) = P(\text{no node in } \mathcal{C}_0(\mathbf{x})) = \exp(-\mu(\mathbf{x}))$. The probability that a node with unknown location is isolated is thus $P(\text{iso}) = \iint_{\mathcal{A}} P(\text{iso} \mid \mathbf{x})\, f_{\mathbf{X}}\, dA$. Such isolation events are 'almost independent' from node to node for small r_0. Thus, the probability that none of the n nodes is isolated is $P(\text{no iso node}) \simeq \exp(-n\,P(\text{iso}))$.

Using numerical integration, we compute $P(\text{no iso node})$ for uniform and RWP nodes on a disk. Fig. 1 gives an example for $n = 500$. By variation of r_0 and n, we find out the critical (r_0, n)–pairs guaranteeing $P(\text{no iso node}) = 0.99$. They are shown in Fig. 2. For comparison, the curve for Poisson distributed nodes without border effects — each node has $\mu(\mathbf{x}) = \mu = r_0^2 \pi / A$, hence $\hat{r}_0(P(\text{no iso node}) = p, n) = \sqrt{\frac{1}{n}\,(\ln n - \ln\ln \frac{1}{p})}$ — is illustrated.

As discussed above, these curves represent lower bounds for $r_0(P(\text{con}) = 0.99, n)$ in the same scenario. The important question is now: How tight is this bound? To approach this question, let us compare the simulation results on $P(\text{no iso node})$ and $P(\text{con})$ in Fig. 1. There is an unignorable difference between $P(\text{no iso node})$ and $P(\text{con})$ at low probability values, but both curves converge for higher probabilities. Let us express this behavior as

$$r_0\,(P(\text{con})\!=\!p, n) = r_0\,(P(\text{no iso node})\!=\!p, n) + \epsilon \qquad (1)$$

with $\epsilon \to 0$ as $p \to 1$, where $\epsilon \geq 0$. A mathematical basis for this phenomenon is given by Penrose's graph–theoretical theorem [7] about the 'longest link of the random minimal spanning tree' (also see [2]). This theorem can be interpreted as follows: In almost all random uniform node placements, with n sufficiently large, the minimum range needed to avoid isolated nodes is equivalent to the minimum range creating a connected network. We therefore state that $r_0(P(\text{no iso node}) = p, n)$ is a very tight bound for $r_0(P(\text{con}) = p, n)$ for *high* probabilities p. As shown by the simulation results in Fig. 2, it is sufficient to compute the ranges $r_0(P(\text{no iso node}) = 0.99, n)$ and use them as very good approximations for $r_0(P(\text{con}) = 0.99, n)$.

The key question arising for calculation of the connectivity with RWP mobility is whether (1) can also be employed for RWP distributions. To answer this question, let us thus briefly re–interpret Penrose's theorem. We regard a given node placement that was generated by a uniform random distribution. Penrose actually says: if we increase r_0 of all nodes simultaneously (starting at $r_0 = 0$), the final transition from unconnected to connected will be, with high probability, due to a previously isolated node that obtains a neighbor — it will not be due to a fusion of two previously separated partitions; those partitions already connected at lower r_0. This statement was shown to be true for uniformly distributed networks with and without border effects [7]. We note that each node has the same isolation probability in a uniform network without border effects. If border effects are present, the isolation probability for nodes close to the border is higher, and it becomes even more likely that the transition from unconnected to connected is due to an isolated node. Using a spatial distribution that shows a monotonically decreasing node density from the middle toward the border of the area, e.g. an RWP distribution, intensifies this effect.

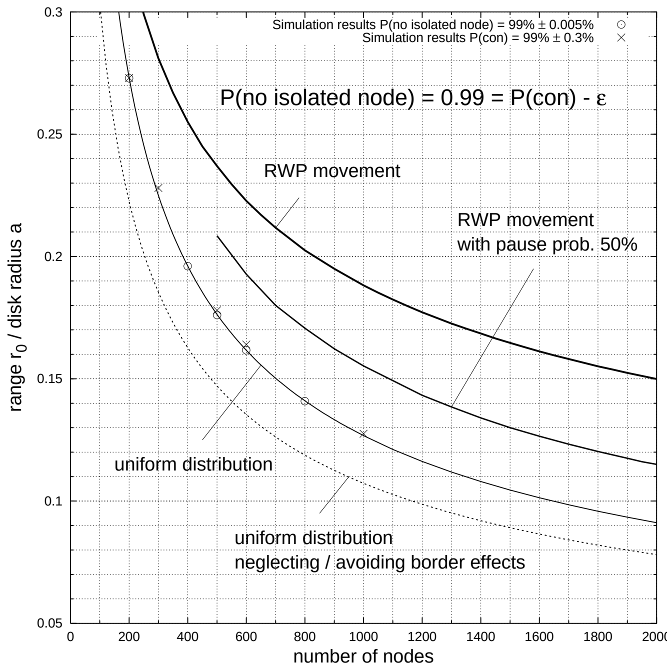

Figure 2: The (r_0, n)–pairs for $P(\textbf{con}) \approx P(\textbf{no iso node}) = 99\,\%$

This discussion leads to the following statement: The critical (r_0, n)–pairs required to keep the network connected during at least $99\,\%$ of the total running time can be well approximated by the critical pairs required to avoid isolated nodes during this time. We computed these pairs for a disk by numerical integration. They are shown in Fig. 2 and can be used by researchers in this field to set simulation parameters accordingly. In short, RWP mobility significantly decreases the connectivity of ad hoc networks compared to uniformly distributed nodes, whereas it increases the expected node degree μ. The shorter $E\{T_p\}$, the lower the connectivity and the higher μ.

Finally, we note that, if the initial spatial distribution of RWP nodes is uniform, and an (r_0, n)–pair is chosen according to Fig. 2, the network is also connected with $P(\text{con}) \geq 99\,\%$ during the 'startup phase' of the mobility model, i.e., when the steady–state spatial distribution is not yet achieved.

5. REFERENCES

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